Proceedings of the American Academy of Arts and Sciences.

Vol. XXXVI. No. 8. — August, 1900.

ON THE THERMAL AND ELECTRICAL CONDUCTIVITY OF SOFT IRON.

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Investigations on Light and Heat made and published wholly or in part with Appropriations from the Rumford Fund.

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Received May 9, 1900. Presented July 17, 1900.

The general method used in the investigation of which this paper is to give an account is set forth with much detail in two articles already published.* Certain more or less important changes of apparatus or procedure will be described and discussed later; but the main results will first be given.

The metal studied was Taylor (Yorkshire) wrought iron, recommended to me by an engineer friend of much experience as the softest wrought iron to be found in the Boston market. Chemical analysis showed the following composition: —

Iron	99.93 %
Carbon	0.059

The density was about 7.785 at 0° C.

The values which I find for the thermal conductivity, k, of this iron are

at 0.1528	28°.2 C.
" 0.1514	58°.3 C.

if the specific heat of water at each of these temperatures is called 1. This would give for the temperature coefficient 0.0003, very nearly.

If the values of k are revised in accordance with the values proposed by Winkelmann † for the specific heat of water at the given temperatures, they become

at 0.1513	28°.2 C.
" 0.1511	58°.3 C.

* Thermal Conductivity of Mild Steel, by E. H. Hall, These Proceedings, XXXI. 271, 1896; On the Thermal Conductivity of Cast Iron, by E. H. Hall and C. H. Ayres, XXXIV. 283, 1899.

† Part 2 of Vol. II. p. 340.

and the temperature coefficient deduced therefrom will be too small to be worth writing down, 0.0000 ?, let us say.

But the recent work of Callendar and Barnes * gives for the specific heat of water

0.9992	at 25° C.
0.9987	" 30° C.
0.9992	" 55° C.
1.0000	" 60° C.

If these values given by Callendar and Barnes are adopted as correct, my first values of k will stand almost without change; and I shall therefore leave them for the present without correction for variation in the specific heat of water.

My value for the temperature coefficient, 0.0003 or a little less, is, so far as it goes, a corroboration of the substantial accuracy of the temperature coefficient found by Lorenz, 0.0002282, although it may be doubted whether the last three figures of this number are of much significance. This agreement is eminently satisfactory to myself; for a comparison of the work of Lorenz with that of other investigators has convinced me that his value of the temperature coefficient is entitled to an especial degree of confidence.

Measurements of the electrical resistance of the iron were made on nine cylinders, each 2 cm. long and about 0.23 cm. in diameter, cut from the same great bar as the disk on which the measurements of k were made. The length of the cylinders, like the thickness of the disk, was taken parallel to the length of the bar; and the cylinders were cut from a part of the bar adjacent to that from which the disk was cut. The extreme difference in the specific resistances of these cylinders was apparently about 5 per cent. The mean specific resistance was found to be 12240 at 18° C. The mean specific conductivity, x, at the same temperature would, therefore, be 817×10^{-7} C. G. S.

The ratio $k \div x$ is about 1716 at 0° C.

The "thermo-electric height" of this iron, as compared with copper, is about

1028×10^{-8}	volts	at	26°.6 C.
980	"	"	"	41°.3 C.
936	"	"	"	54°.5 C.
870	"	"	"	71°.1 C.

* Physical Review, April, 1900.

The relation of the values here given for k and x to those found by others who have studied the thermal and electrical conductivities of soft iron, I have set forth in the "Physical Review" for May–June, 1900.

The following details of my work are perhaps unnecessarily extended and tedious; but a considerable study of the literature of thermal conductivity has convinced me that most experimenters in this field have omitted important matters in the printed description of their investigations. It is my hope that those who may have to deal with problems similar to, though not exactly like mine, will find what is here written worthy of their attention.

The iron used for the experiments on thermal conductivity was in the form of a disk cut from the end of a five-inch cylinder and turned down at first to a diameter of 10.5 cm. The thickness of the disk was about 1.996 cm., the greatest thickness indicated by the calipers being 1.998 cm., and the least 1.995 cm.

Treatment of the Disk.

The disk was coated with copper electrolytically on both faces by a method substantially the same as that previously described. Spots which, because of slight flaws in the surface, appeared not to be taking the copper well from the preliminary cyanide bath, were rubbed with the point of a lead pencil to give them a coating of graphite, after which they speedily became coppered like the rest. In the sulphate bath the convex surface of the disk was protected as before by rubber bands; but outside these bands was now placed a band of paraffined paper about 5 cm. wide, the object of which was to impede the deposit at the edge of the faces of the disk and so make it keep better pace with the rate of deposit at the centre of the faces. As before, it was occasionally necessary during the progress of the deposit, which lasted about a week, to remove the disk from the bath in order to break off or file off projecting pimples, or *corals*, of copper.

The final coating of copper on each face, after being turned down nearly to a plane, was about 0.2 cm. thick. The whole curved surface was now turned down until the diameter of the disk was 10.00 cm.

Mounting and Use of Disk and Adjacent Apparatus.

Figures 1 and 2 of an article already mentioned, "On The Thermal Conductivity of Cast Iron," indicate with accuracy in most particulars

the method of mounting and using the disk now under consideration. Certain small changes, however, must be imagined in Figure 2, in order to make it accord perfectly with the latest developments of the apparatus. Thus, the water stream entering beneath the disk is no longer allowed to flow without restriction straight against the centre of the lower face. A little affair shaped somewhat like a three-legged stool is placed at the top of the admission tube, just beneath the point marked by the letter C′, and most of the water escapes laterally between the legs of the stool, although a small part of it runs through a hole in the top straight against the disk. This device was adopted in the hope that it would make the temperature more uniform over the lower face of the disk. With a similar purpose regarding the upper face of the disk, the hard rubber block HH has been replaced by one having a somewhat more gradual curvature, so as to make a thinner and more rapid stream over the central parts of the disk. It is doubtful whether these changes have done much good, on the whole, although they appear to have made the difference of temperature between top and bottom of disk at the centre very nearly equal to the mean difference at other parts, as numbers presently to be given will show. Another attempt to improve the distribution of temperature, by making the flow of water more nearly equal along different radii of the upper face of the disk, affected the manner of admission of the water to the funnel FF; but, as it was of doubtful utility, it need not be described. The air-vent leading up from the funnel FF by the tube *w* has been considerably enlarged; but the small escape of water which had previously been maintained at this vent is no longer permitted.

The small copper wires, extending from the copper coatings of the disk, are now protected from actual contact with the hard rubber plugs K_1, K_2, etc., and with the soft rubber packing surrounding these plugs, by a wrapping of oiled silk, with the purpose of preserving the wires from the destructive action of the sulphur emitted from the rubber; but, after all, the wires cannot be depended upon for more than a few months.

Stops about 0.16 cm. thick, placed at the edges of the faces of the disk, prevent the blocks HH and H′H′ from approaching these faces so near as to endanger the safety of the small wires or cut off the flow of water.

The parts marked J_1 and J_2 in Figure 2 of the former paper now contain spirals of platinum instead of thermo-electric junctions of copper and German silver, change of resistance of platinum having been substituted for change of thermo-electromotive force as a means of measuring

the change of temperature of the stream of water which flows over the upper surface of the disk. More will be said of this later. The parts surrounded by the water-jacket are carefully and thickly wadded with cotton-wool so as to make a nearly cylindrical body, about 18 cm. in diameter, up to the plugs J_1 and J_2. The plugs also are covered with cotton-wool, as well as the slot in the top of the jacket. Around the disk itself the wadding is so thick as barely to allow the jacket to enclose it.

The copper wires leading out from the plugs J_1 and J_2 ran, after July 17, outside the cotton-wool wrapping, but without touching the jacket.

Determination of the Difference of Temperature of the Two Faces of the Disk.

As before, this is effected by thermo-electric means, the iron disk and its two copper coatings being used as a thermo-electric couple. As before, thirteen fine copper wires lead off from as many points on the upper coating, and similar wires from corresponding points on the under coating. Each pair of corresponding wires can be used singly, or all the thirteen pairs can be joined and used in multiple by an arrangement described in a preceding paper. The distribution of wires over either coating is shown in Figure 1, the numerals being alongside the points of attachment to the coating.

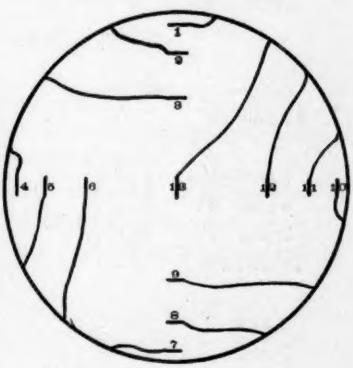

Figure 1.

In the arrangement of these points there was an attempt to make the various areas represented respectively by the individual points as nearly equal as practicable. Point 13 is intended to be at the centre of the disk; points 3, 6, 9, and 12, 2.55 cm. from the centre of the disk; points 2, 5, 8, and 11, 3.80 cm. from the centre of the disk; points 1, 4, 7, and 10, 4.60 cm. from the centre of the disk.

No great accuracy is attained in placing these points; and, in fact, each attachment is rather a line, about 0.5 cm. long, than a point, each such line, except No. 13, crossing nearly at right angles the radius upon which it lies.

It is, of course, desirable to make the flow of water along the two faces of the disk such that the indication received from any one pair of wires, one above and one below, shall be about the same as that given by any other such pair of wires, and much thought has been given to the attainment of this end. The result is not quite all that could be wished for; but it is such that any important error from the inequalities observed is very unlikely. The following tables show the results of tests made under conditions as nearly uniform as it was found practicable to keep them. The galvanometer deflection credited to each pair of junctions is the mean obtained from two short sets of observations, the various pairs being used first in the order in which they are here given, and then in the reverse order. Between these forward and back series of observations with single pairs a short set of observations with all the pairs in multiple was made, and the mean deflection from this set, corrected for the difference of resistance of the multiple and single arrangements, is also given below.

Temperature about 28° C.

Junctions.	Deflections.	
13 and 13′	11.1	
1 and 1′	11.0	11.6
4 and 4′	11.1	
7 and 7′	12.0	
10 and 10′	12.2	
2 and 2′	10.9	11.3
5 and 5′	10.8	
8 and 8′	11.7	
11 and 11′	11.6	
3 and 3′	10.6	10.7
6 and 6′	10.7	
9 and 9′	10.5	
12 and 12′	11.1	
Mean,	11.2	

Along radii		
1 and 1′	11.0	10.8
2 and 2′	10.9	
3 and 3′	10.6	
4 and 4′	11.1	10.9
5 and 5′	10.8	
6 and 6′	10.7	
7 and 7′	12.0	11.4
8 and 8′	11.7	
9 and 9′	10.5	
10 and 10′	12.2	11.6
11 and 11′	11.6	
12 and 12′	11.1	

All in multiple, 11.3

Temperature about 57° C.

Junctions.	Deflections	
13 and 13′	10.4	
1 and 1′	11.5	10.7
4 and 4′	9.5	
7 and 7′	10.8	
10 and 10′	10.8	
2 and 2′	11.5	10.7
5 and 5′	9.5	
8 and 8′	10.6	
11 and 11′	11.3	
3 and 3′	11.0	10.3
6 and 6′	9.7	
9 and 9′	9.9	
12 and 12′	10.6	
Mean,	10.5	

Along radii.		
1 and 1′	11.5	11.3
2 and 2′	11.5	
3 and 3′	11.0	
4 and 4′	9.5	9.6
5 and 5′	9.5	
6 and 6′	9.7	
7 and 7′	10.8	10.4
8 and 8′	10.6	
9 and 9′	9.9	
10 and 10′	10.8	19.0
11 and 11′	11.3	
12 and 12′	10.6	

All in multiple, 10.4

It seems likely that the differences between the different radii are due in part to inequalities of water-flow caused by the lodging of air-bubbles at various points on or near the surfaces of the disk.

The agreement between the deflection obtained with all junctions in multiple and the mean of those obtained with the pairs used singly appears satisfactory, in view of the not very rigid character of the test by which the comparison is made.

The resistance of each individual pair of wires, out to the point where all were connected in multiple, is shown in the following table, as it was found July 12th, 1899: —

13–13′	0.56 ohm	8– 8′	0.56 ohm
1– 1′	0.55 "	11–11′	0.56 "
4– 4′	0.55 "	3– 3′	0.55 "
7– 7′	0.56 "	6– 6′	0.56 "
10–10′	0.56 "	9– 9′	0.55 "
2– 2′	0.55 "	12–12′	0.56 "
5– 5′	0.56 "		

The deflections noted above are those of a rather sensitive astatic galvanometer, the sensitiveness of which was frequently determined by means of a potentiometer and a standard Carhart cell, or rather, two

such cells, which differed from each other in electromotive force about 1 part in 500.

The mean difference of temperature of the two streams of water on entering the apparatus was about 8° at low temperatures and about 7°.6 at high temperatures. The mean difference of temperature between the two surfaces of contact of the iron and copper, as indicated thermo-electrically, was about 1°.42 at low temperatures and about 1°.58 at high temperatures, which shows that heat is communicated more readily from the water to the copper coatings, and *vice versa*, at high temperatures than at low temperatures, other things being equal.

In order to determine the difference of temperature, just mentioned, between the surfaces of contact of the copper with the iron disk, it was of course necessary to find the thermo-electromotive force corresponding to a known difference of temperature of two similar junctions. This datum was obtained by the method described in Appendix I of the paper on the "Conductivity of Cast Iron," to which paper a number of references have already been made in this writing. Some particulars of the present case follow.

A second disk about 2.5 cm. thick was cut from the same end of the same cylinder of Yorkshire iron that had furnished the disk already described. This second disk was then cut into two half disks, and from one of them was taken a slice, thickness-wise, about 11 cm. long, 2.5 cm. wide, and 0.3 cm. thick. This slice was then sawed up, crosswise, into thirty-three bars. All but ten of these bars were then reduced by filing and milling to a diameter of about 0.16 cm., and the length of each was reduced to 2.0 cm. The remaining ten were reduced in the same way to a thickness of 0.23 cm. and a length of 2 cm. The bars were then numbered from 1 to 33 in the order of their original position in the slice from which they had been cut. Numbers 1, 4, 7, 10, 13, 20, 23, 26, 29, and 32 were placed end to end, in the order just given, in the bore of the wooden cylinder of the thermo-electric test apparatus shown in Figure 5 of the article on "Conductivity of Cast Iron." No essential change has been made in this apparatus or in the manner of using it since the description given in the article just mentioned was written, except in these particulars, that the electrical resistance * of the end-to-end row of bars

* This resistance was likely to be as much as 2.5 or 3 ohms when the pressure, about 3 kgms., was newly applied to the row, but a gentle rocking, from side to side, of the copper blocks, kept well seated, would in the course of a few minutes reduce this resistance to less than 1 ohm, the end pressure remaining un-

and copper end-pieces has been dealt with more carefully and successfully in recent experiments than in earlier ones, and that in some cases the bars have been placed in a glass tube instead of a wooden one. The results of these thermo-electric tests, subject to slight corrections for peculiarities of the thermometers * used, are given below. The temperature put down for each case, in the second column, is the mean of the thermometer readings in the two copper blocks at the ends of the row of iron bars. The third column, headed Δ, gives for each case the mean difference of temperature of the two thermometers. The fourth column, headed E, gives for each case the thermo-electromotive force corresponding to a difference of 1° between the two thermometers, each of which thermometers is supposed to indicate with sufficient accuracy the temperature of the copper-iron contact neighboring to it.

Date	T		Δ		E	
June 3, '99	25°.8		7°.04		1041×10^{-8}	
" 7, "	27°.7		7°.52		1017 "	
" 30, "	25°.3		7°.61		1024 "	
July 3, "	26°.5	26°.6	8°.22	7°.30	1042 "	1028×10^{-8} volt
" 4, "	26°.4		6°.64		1031 "	
" 6, "	27°.8		6°.79		1012 "	
June 6, "	40°.5		7°.44		979 "	
" 7, "	44°.6		7°.23		964 "	
July 3, "	40°.6	41°.3	7°.79	7°.19	982 "	978×10^{-8}
" 4, "	38°.8		6°.94		990 "	
" 6, "	42°.0		6°.57		976 "	
June 6, "	55°.6		7°.03		926 "	
" 7, "	56°.5		6°.95		930 "	
July 3, "	52°.2	54°.5	7°.60	6°.89	941 "	934×10^{-8}
" 4, "	53°.8		6°.64		938 "	
" 6, "	54°.6		6°.25		933 "	
June 3, "	69°.0		6°.07		885 "	
" 7, "	74°.5		6°.24		850 "	
" 30, "	69°.4		6°.18		865 "	
July 3, "	70°.6	71°.1	7°.04	6°.37	874 "	871×10^{-8}
" 4, "	71°.3		6°.04		879 "	
" 6, "	71°.9		6°.66		870 "	

changed, and, except during the moments of measurement of resistance, no electric current flowing through the row of bars.

* Baudin, Nos. 10,286 and 10,287, as in previous work.

Applying to the mean values of E, found above, certain small corrections which take account of errors in the graduation of the thermometers, we get, —

T	E
26°.6	1028×10^{-8} volt
41°.3	980 " "
54°.5	936 " "
71°.1	870 " "

These numbers plotted, with temperatures for abscissas and electromotive force per degree for ordinates, indicate a curve which descends with very gradually increasing slope with rise of temperature. Indeed, this curve is so nearly a straight line that its curvature cannot be satisfactorily shown in a small figure. It would be almost perfectly straight if the numbers given under E were 1028, 975, 929, and 870.

The minute examination which I was obliged to give to the individual small cylinders, during measurements of their electrical resistance, led me to notice defects and possible distortions which might, I feared, have affected their thermo-electric quality. Accordingly, several months after tests just described were made, I undertook a similar test with ten of the somewhat larger cylinders already mentioned, which had apparently suffered much less in the process of milling. In this later test I found it convenient to enclose the bars in a tube of glass instead of a tube of wood. This test gave 1064×10^{-8} for E at 15°.4 C.

The earlier tests, above described, did not run so low in temperature, but by extrapolation they give, for 15°.4 C., $E = 1065 \times 10^{-8}$, or something very close to that, a satisfactory agreement.

Determination of the Difference of Temperature of the Ingoing and Outgoing Water at the Chamber above the Disk.

It has been already stated that a differential platinum thermometer was used for this purpose instead of the two copper-German-silver thermoelectric junctions which had been employed in the preceding investigations. This change was the result of a conviction, the fruit of much experience and vexation, that no suitable permanent protection can be found for the thermo-electric junctions against the action of hot water. A thin layer of shellac well dried on appears to be the best coating; but this is liable to give way at a most inconvenient time and bring to naught

the labor and observations of hours. The platinum thermometer method is not without difficulties, as the following pages will show; but it appears preferable to the other.

Figure 2 represents one of the platinum spirals, S, in position for use. It consists of about 22 cm. of wire, 0.012 cm. in diameter. The diameter of the coils is about 0.45 cm. The ends of this platinum wire are soldered

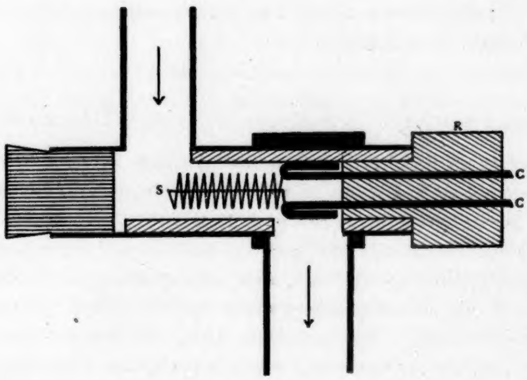

FIGURE 2.

to copper wires, C and C, each about 1.6 m. long and 0.1 cm. in diameter, which extend through the hard rubber plug R. Each of the copper wires is soldered at the outer end to a copper rod 5 cm. long and 0.6 cm. in diameter, which is well amalgamated at the free end and serves to make connection with a mercury well of a Carey Foster bridge. Care was taken to make the length of the wire very nearly equal in the two spirals, and equal care to make the copper wires, all of which are from the same piece, all alike at first. The parts of Figure 2 which are in solid black represent metal; the parts cross-hatched thus / represent soft rubber.

The resistance of each spiral was about 1.4 ohms, and that of its connecting copper wires about 0.1 ohm. Trial showed that one spiral with its connecting wires had a resistance about 0.0006 ohm greater than that of the other spiral and its connections. Slight changes in the copper wires reduced this difference of resistance to something like 0.000025 ohm. As the original difference of resistance was probably nearly all in the spirals, these continued to differ, at the temperature of the room, by something like 0.0006 ohm, which would correspond to about one-seventh part of a degree difference of temperature. No attempt at a

closer adjustment of the spirals was made, as the method of experimentation was expected to eliminate from the result any considerable error arising from this inequality. That this expectation was finally justified will be shown later, although more difficulty was encountered than at first appeared, the fact being, apparently, that the two spirals, although made from the same piece of wire, did not have quite the same temperature coefficient of resistance, so that the difference of resistance between them was not constant when their temperature was varied,* but increased when the spirals were heated.

Calibration of Differential Platinum Thermometer.

In order to calibrate this differential platinum thermometer, the two spirals were placed in streams of water, the temperatures of which were measured by means of the same thermometers that were used in studying the copper-iron thermo-electric junctions, and the difference between the electric resistance of spiral No. 1, with its connecting wires, and that of spiral No. 2, with its connecting wires, was measured by means of a Carey Foster bridge. The conditions under which this trial was made resembled very closely those under which the spirals were to be used in the main experiment, the plugs bearing the spirals being inserted in the same sockets in which they were afterward to be used, which sockets were temporarily removed from the main apparatus and fitted to brass tubes through which flowed the streams of water employed for the test; these tubes were wrapped around with cotton wadding, and sockets and tubes together were surrounded by the same water-jacket that was used later to surround the main apparatus. The copper connecting wires led down between the cotton wadding and the water-jacket, and then out beneath the latter without touching it; this detail is mentioned because the temperature of the connecting wires is by no means a matter of indifference in some parts of the investigation. In this calibration test each stream flowed past the thermometer bulb before reaching its spiral, and past the spiral before reaching the ends of the copper wires which carried the spiral. The distance from each thermometer bulb to the

* The wire from which the spirals was made was annealed by drawing it through a flame, which treatment may have introduced into it some lack of uniformity. After the spirals were formed and soldered to the copper they were kept in a bath of melted paraffine, in the neighborhood of 145° C., about two hours, with the object of removing inequalities of condition caused by the bending to which the wire had been subjected.

spiral beyond it was perhaps 2 cm. The velocity of the stream between the bulb and the spiral was some 30 or 40 cm. per second. One stream was usually about 6°.7 warmer than the other. The method of alternation was used; that is, if the stream passing spiral No. 1 was during one set of observations kept a certain number of degrees warmer than the other stream, the streams were exchanged at the end of that set and another set was then made, the difference of temperature and the mean temperature remaining nearly as before, and no part of the apparatus suffering change of place except the cock by means of which the change of flow was affected. The combination of the two complementary sets of observations gave a result from which errors due to disagreements of the thermometers and lack of perfect equality of the spirals was practically eliminated. Sets of observations were made at various mean temperatures, and, in order to make the results at these various temperatures comparable, slight corrections, amounting at the most to less than one-half of one per cent, were made in certain cases because of errors in the graduation of the thermometers, the combined diameters of the bores being slightly greater at some temperatures than at others. Similar corrections made in calibrating the copper-iron thermo-electric junctions have already been referred to.

Each of the horizontal lines, numbered from 1 to 8 in the table below, gives the result of two complementary sets of observations, such as have been described above. T is the mean temperature of each pair of sets; Δ is the mean difference of temperature of the streams; L is the mean length of bridge wire included between the two points of equilibrium corresponding to the two positions of the commutator of the Carey Foster bridge.

Date	T	Δ	L	L ÷ Δ	
May 5, 1899	20°.77	6°.68	20.30 cm.	3.039	(1)
" 6, "	36°.72	6°.33	19.21 "	3.032	(2)
" 11, "	70°.41	6°.55	19.86 "	3.032	(3)
" 11, "	21°.32	6°.99	21.20 "	3.033	(4)
" 12, "	76°.12	6°.63	20.21 "	3.048	(5)
" 17, "	51°.60	6°.80	20.57 "	3.024	(6)
" 26, "	52°.46	6°.96	21.01 "	3.018	(7)
" 26, "	37°.69	7°.15	21.59 "	3.019	(8)

The mean resistance of the bridge wire used in this test was 0.001491 ohm per cm., the material being German silver and the diameter about 0.2 cm. The mean resistance of the bridge wire, German silver about 0.4 cm. in diameter, employed in later experiments, when the spirals

were used to determine the change of temperature of water in the heat-conduction apparatus, was 0.0002548 ohm.* With these data we get from the table above: —

Mean of	T	L ÷ Δ	l
(1) and (4)	21°.1	3.036	17.77
(2) " (8)	37°.2	3.026	17.71
(6) " (7)	52°.0	3.021	17.68
(3) " (5)	73°.3	3.040	17.79

Here l stands for $(L \div \Delta) \times (0.001491 \div 0.0002548)$, that is, the length of bridge wire, used in the main experiments, corresponding to a difference of 1° in the temperature of the spirals. Values of l for temperatures intermediate between those here given were found when needed by interpolation between the values of l here given. It will be observed that there is a maximum difference of 11 parts in 1770 between the given values of l. Rigid constancy of l would mean rigid constancy of the mean of the temperature coefficients of the two spirals with reference to the mean scale of the mercury thermometers used. Such constancy was hardly to be expected.

Use of Differential Thermometer in the Main Experiments.

In the main experiments the plugs bearing the spirals were at first placed in their supporting sockets according to Figure 2, which represents the spiral placed in the incoming stream. The water, therefore, on entering the apparatus, passed the first spiral before reaching its copper connections, but on leaving the apparatus it passed the copper connections of the second spiral before coming to the spiral itself. Accordingly, any net conveyance of heat by the copper wires into or out of the stream was superposed on the action of the conducting disk to produce the difference of temperature noted by the spirals. Any such effect of the copper wires would probably be very small except at high temperatures; and at such temperatures the effect would be, in large measure, eliminated from the result by the practice, always maintained, of combining one set of observations in which the disk carried heat to the upper stream with another set of observations in which the disk carried heat from the upper stream, the mean temperature of the disk re-

* Each bridge wire was calibrated by finding what length upon it corresponded to the difference between the resistance of a 1 ohm coil and that of a similar coil shunted by a known, much larger, resistance.

maining nearly unchanged throughout the two sets.* The elimination, however, would not be perfect; for the reason that the temperature of the stream which we are considering is not quite the same in the two complementary sets of observations just described, being about 8° warmer in one set than in the other. The question at issue comes, therefore, very nearly to this, whether the amount of heat carried away from the stream by the wires, when the stream is 8° warmer than the room, is negligible in comparison with the amount carried away by the disk, the difference of temperature of the two faces of the iron being, as we have seen, usually 1°.4 or more. This question can be answered in the affirmative; for the aggregate cross-section of the four copper wires is little, if any, more than 0.03 sq. cm., and the length of each wire, from the spiral out to the point where it is exposed to the temperature of the room, is not far from 30 cm. The carrying power for heat of a rod of copper 30 cm. long and 0.03 sq. cm. in cross-section, the thermal conductivity of copper being taken as eight times that of iron, would be about one five-thousandth part of the carrying power of the iron disk for a given difference of temperature, and not more than one eight-hundredth part, if the difference of temperature were 8° for the rod and 1°.4 for the disk. Nevertheless, after a considerable number of trials had been made with the spirals placed as in Figure 2, the plugs bearing the spirals were put in at the other ends of the supporting sockets, so that any changes of temperature produced in the stream by the copper wires must now occur either before or after the change of temperature noted by the spirals. The mean of the results obtained after this change of arrangement was slightly different from the mean of those obtained before; but it is unlikely that the difference was due to this change.

In the observations of July 31 and thereafter a loop of each copper wire was kept in a pocket containing oil on the outside of the water-jacket, so that each wire had at this point a temperature not more than one or two degrees different from that of the spiral with which it was connected.

Another question was whether the heat carried in or out by the copper wires affected the temperature of the spirals directly by metallic conduction, so as to keep them at a temperature different from that of the water passing them. The reasons for thinking that any such effect was negligible are given in the Appendix to this paper.

It has been stated that there was a difference of resistance between the two spirals at any given temperature, and that this difference in-

* The time between the exchange of the water streams and the beginning of the next set of observations was usually rather more than twenty minutes.

creased with rise of temperature. This difference, if constant at each temperature, would be eliminated by combining two sets of observations made at the same mean temperature, one set being made with spiral No. 1 the warmer, and the other with spiral No. 2 the warmer. Sets of observations were, it is true, combined in pairs, but in each such pair it was the mean temperature of the disk that was kept nearly constant, while the mean temperature of the spirals was seven or eight degrees warmer in one set than in the complementary set. Accordingly, the mean result of a pair of sets, taken without individual correction for the inequality of the spirals, would have been subject to the error caused by ignoring the variation of this inequality through a rise of, we will say, 8 degrees in mean temperature. This error would have been about 1.5 per cent of the final result at any temperature. To prevent such an error, corrections, based on careful observations made for this specific purpose, were applied in each individual set of observations for conductivity; and, after these corrections had been applied, the complementary sets were put together for a mean result at any given temperature of the disk. By this means the error in question was probably reduced to very small dimensions.

It appeared likely that some part of the apparent difference of resistance of the spirals at high temperatures was due to difference of resistance of the copper connecting wires, which near the spirals and for some distance away from the latter were considerably heated. As the experiments went on, increasing care was taken to make the condition of the wires leading to one spiral as nearly as might be the same as the condition of those leading to the other spiral, in order that the differential changes of resistance should be confined to the spirals themselves. The result was a progressive diminution of the correction for isothermal inequality of resistance at high temperatures; but this correction remained large and somewhat uncertain to the end, so that the value of the conductivity calculated from any one set of observations, unbalanced by its complementary set, was liable to a considerable error, as the details presently to be given will show. If I were to go through such work again, I should try to reduce the importance of temperature changes in the copper connecting wires by increasing the resistance of the spirals or the change of temperature of the stream between them.

Heating and Flow of Water.

The method of heating and controlling the flow of the streams of water has been described in previous papers. The only feature of importance

to add to the description is this, — that in these later experiments the freedom of movement of the covers of the gas-holders, upon which the accuracy of regulation of the gas pressure depends, has been greatly increased by keeping the hammer of an electric bell in brisk action against the side of each gas-holder. When the regulating devices were working well the maximum variation of temperature in either stream during one of the main sets of observations, occupying some 45 minutes, was usually about one fifth of a degree C.

Results in Detail for Thermal Conductivity.

Experiments on conductivity of the iron with the apparatus as described in this paper were begun July 13, 1899. The results of that day and of July 14 are not here given, as they were obtained before the full importance of some precautions was recognized; but all the results obtained later are recorded below. The subscript 1 refers to cases in which the warmer stream of water ran above the disk, the subscript 2 to those in which it ran beneath the disk. T, the mean of T_1 and T_2, is the mean temperature of the disk during a pair of complementary trials. K, the mean of K_1 and K_2, is the mean value of the conductivity given by a pair of complementary trials. The results obtained July 31 and August 2 are distinguished from the others by the fact that they were made after a certain change in the position of the thermo-metric spirals (see p. 135).

Date	T_1	T_2	K_1	K_2	T	K	
July 17, '99	29°.2	28°.9	0.1521	0.1531	29°.1	0.1526	
" 18, "	28°.7	28°.5	0.1468	0.1553	28°.6	0.1511	
" 22, "	28°.2	28°.1	0.1568	0.1500	28°.2	0.1534	0.1528
" " "	28°.1	27°.8	0.1539	0.1513	28°.0	0.1526	
" 27, "	27°.4	27°.5	0.1580	0.1510	27°.5	0.1545	
" 31, "	27°.7	27°.4	0.1465	0.1529	27°.6	0.1497	0.1528
Aug. 2, "	28°.3	28°.2	0.1543	0.1573	28°.3	0.1558	
			0.1526	0.1530	28°.2	0.1528	
July 18, "	58°.8	58°.6	0.1531	0.1529	58°.7	0.1530	
" 22, "	58°.9	58°.8	0.1608	0.1441	58°.9	0.1525	0.1521
" " "	58°.6	58°.7	0.1593	0.1420	58°.7	0.1507	
" 27, "	56°.2	57°.6	0.1552	0.1490	56°.9	0.1521	
" 31, "	57°.8	57°.7	0.1510	0.1498	57°.8	0.1504	0.1502
Aug. 2, "	58°.3	58°.6	0.1568	0.1432	58°.5	0.1500	
			0.1560	0.1468	58°.3	0.1514	

The values of K given above take no account of the variation in the specific heat of water between 28°.2 and 58°.3. The temperature coefficient of K obtained from the trials made July 31 and August 2 after a change in the arrangement of the spirals, which change was supposed to make for greater accuracy of results, is decidedly greater than that obtained from the trials which preceded this change; but the considerable difference between the values of K at low temperature found July 31 and August 2 makes it unsafe to give especial weight to the value of the temperature coefficient calculated from the trials of these two days. The best course appears to be to take the mean of all the results at low temperature and compare it with the mean of all those at high temperature; and this has been done, with the result stated at the beginning of this paper.

Measurement of the Electrical Resistance.

A satisfactory determination of the mean electrical resistance of the iron was a work of considerable difficulty. I attempted it at first by use of several little cylinders, each taken singly, such as had been used in testing the thermo-electric quality of the iron (see p. 128). I found, however, that the last milling which these cylinders had been subjected to had left them somewhat irregular in diameter, so that it was impossible to measure this dimension accurately, even when calipers with jaws meeting along a narrow line were used. Accordingly I used the somewhat larger cylinders, already mentioned, which had suffered much less from the imperfections of the milling process. These were about 0.23 cm. in diameter, and could be measured with satisfactory accuracy.

The straight thick wire of a Carey Foster bridge having been replaced by two stout brass rods lying in line, with a gap between their ends, which were amalgamated, one of the iron cylinders was placed end to end between these rods and firmly held there under considerable pressure. Then the resistance of a certain measured length, 1.472 cm., of the iron bar was determined in the usual way by means of the bridge and the accompanying low resistance coils. Four cylinders were tested in this way, and their mean specific resistance was found to be 12430, C. G. S., at 22° C.

Later the resistance of the same length of each of these bars and of five other similar bars, held as just described, was measured by a potentiometer method, the potentiometer wire being of copper drawn especially for this test. The mean specific resistance of the nine bars, as found by this latter method, was 12240, C. G. S., at 18° C., the value given in the beginning of this paper.

APPENDIX.

Does the conductive action of the copper wires attached to the platinum thermometer spirals introduce error by preventing these spirals from taking the temperature of the water flowing past them?

If the warming or cooling produced by the action of the copper wires were equally great at the two spirals, no harm would result, as it would not affect the difference of their temperatures, which is the quantity measured; but this perfect compensation can hardly be, for all of the copper connections are subjected to very nearly the same temperature conditions outside the supporting hard rubber plugs, while the spirals themselves differ in temperature about $0^\circ.5$. The differential effect, upon which alone the possibility of sensible error depends, is very much the same as if one of the spirals were at the same temperature as its connecting wires outside the plug and the other spiral $0^\circ.5$ warmer or colder than its connecting wires outside the plug. It is possible to make a very rough estimation of the maximum amount of error which could arise from such a condition. For the purpose of this calculation it may be assumed that the hard rubber plug is a non-conductor of heat, — an assumption which tends to magnify the effect under discussion. The length of the plug, that is, the length of the wire from air to water, is 2.5 cm., the diameter of the wire about 0.1 cm., the length of each copper wire exposed to the water about 1.6 cm. The length of the platinum wire in each spiral is about 22 cm., and its diameter about 0.012 cm. The point of attachment of the platinum to the copper is near the middle of the part of the copper exposed to the water. We will discuss the action of a single copper wire, and assume that one half of the platinum wire was attached to this by one end, the other end being free. For this purpose it will be necessary to know something about the "surface conductivity" of copper immersed in running water. Fortunately the main experiment with the copper-coated disk gives us some information in regard to this, — very inaccurate information, no doubt, but sufficient for the present purpose.

In this main experiment, with a mean temperature t in the disk, and with a temperature $t + 4$ in the stream on one face and $t - 4$ in the stream on the other face, the two meeting surfaces of copper and iron had respectively temperatures about $t + 0.8$ and $t - 0.8$, we will say. Assuming the thermal conductivity of copper to be eight times that of iron, and remembering that the copper coatings are each 0.2 cm. thick while the disk is 2 cm. thick, we get for the temperatures of the two outer

copper surfaces, in contact with the water, the temperatures $t + 0.82$ and $t - 0.82$ respectively. This makes each copper coating to have a temperature gradient of 0°.1 per cm., with a difference of 3°.18 between its outer surface and the water stream flowing across it. The ratio of the temperature gradient to the external difference is therefore about $\frac{1}{32}$.

According to this we may infer that a stream of water flowing across one face of a copper wire, with a speed equal to that of the flow across the surface of our disk, and with a temperature t degrees above the temperature of the face of the wire, will maintain within that wire a gradient of temperature equal to $t \div 32$, all lateral action being excluded.

The point of attachment of the platinum wire to the copper is about midway of the exposed part of the copper, and is as much as 3.0 cm. from the outer end of the plug. If the copper wire terminated at this point of attachment, and suffered conductive contact with the water only at its terminal surface, the change of temperature from the outer end of the plug, supposed non-conductive, to the end of the wire would accordingly be about $\frac{3}{32}$, $\frac{1}{11}$, as great as the difference of temperature between the end of the wire and the water flowing past it. If, therefore, the wire at the outer end of the plug exceeds in temperature the stream of water by 0°.5, as we will assume, the fall of temperature within the wire would be about 0°.04, and the end of the wire would be about 0°.46 above the temperature of the stream. This conclusion, however, is based on a false assumption as to the area of contact of the wire with the water; in fact, this area of contact is about sixty times as great as the cross-section of the wire, and the point of attachment of the platinum is near the middle of this area, so that we shall not be very far from the truth in assuming that the temperature of the copper at the cross-section next the point of attachment of the platinum is the same that it would be if the wire had contact with the water only at this cross-section but had sixty times as great a surface conductivity as such an area really has in contact with the stream. This leads to the conclusion that the fall of temperature within the wire, from the outer end of the plug to the point of attachment of the platinum, is about $\frac{60}{11}$ times as great as the difference of temperature between the point of attachment and the stream. This last difference would therefore be rather less than 0°.1, but we will call it that.

The problem now is to find how much the mean temperature of a platinum wire 0.012 cm. in diameter and 11 cm. long will exceed that of the water stream in which it is placed, if one end of this wire is kept 0°.1 above the temperature of the water. This problem is of a familiar sort,

and is easily dealt with if we know the ratio between the thermal conductivity, K, of the platinum and its "surface emissivity," E, in the stream of water. Assuming E to be the same for platinum as for copper, and K to be $\frac{1}{8}$ as great for platinum as for copper, we get for $E \div A$ the value $\frac{1}{4}$. We now have (see Preston's "Heat," p. 513)

$$\mu^2 = \frac{E\,p}{K\,A} = \frac{1}{4} \times \frac{.012}{.006^2} = 83,$$

or $\mu = 9$ in round numbers. Then, using the formula $\theta = \theta_0\,\epsilon^{-\mu x}$, where θ_0 is excess of temperature of the heated end of wire above temperature of water, θ the excess of temperature at any point distant x cm. from this end, and ϵ the Napierian base, we have, reckoning θ in per cents of a degree: —

x	θ		
0	10		
		>7.1	
0.1 cm.	4.1		
		>2.8	
0.2 "	1.7		
		>1.2	Mean 2.4 % of 1°
0.3 "	0.7		
		>0.5	
0.4 "	0.28		
		>0.2	
0.5 "	0.11		

Accordingly the mean excess of temperature of the wire along its first 0.5 cm., less than $\frac{1}{20}$ of its whole length, would be about 5 % of the usual difference of temperature, about $0^\circ.5$, between the two spirals; and beyond this point the excess would be very small; so that the error made by neglecting the difference of temperature between the spirals and the water is not important, provided the calculation just made is tolerably accurate. The most uncertain element in this calculation is probably the value of the "surface emissivity," which is based on observations made on the behavior of the disk and the streams across its face. But as the velocity of the water in passing the spirals is probably ten times as great as its mean velocity across the disk, it seems altogether likely that a sufficiently low estimate of emissivity has been used in the calculation, and that the possible error from the source in question has been overestimated in the discussion just given.